**BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI**

**First Semester, 2020-21**

**MATH F444, Numerical Solutions to Ordinary Differential Equations**

**Assignment 2 Date: 17/11/2020**

**Name: Salmaan Shahid ID no: 2016B4A70580P**

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Gas flows through micro/nano scale channels find their applications in broad areas of science, engineering and bio-medical sciences. We consider processes that fall into the class of steady shear flows, mainly steady Poiseuille flows [[M. Torrilhon and H. Struchtrup](https://doi.org/10.1016/j.jcp.2007.10.006)].

Let us consider shear flow which is homogeneous in direction and for the velocity we assume that thus the velocity vector is given by

The flow is driven by a body force (gravity or a pressure gradient) acting only in direction,

This setting is valid for channel flows as displayed in Fig. 1. The fluid (ideal gas) is confined between two infinite plates at distance and is moving solely in direction.

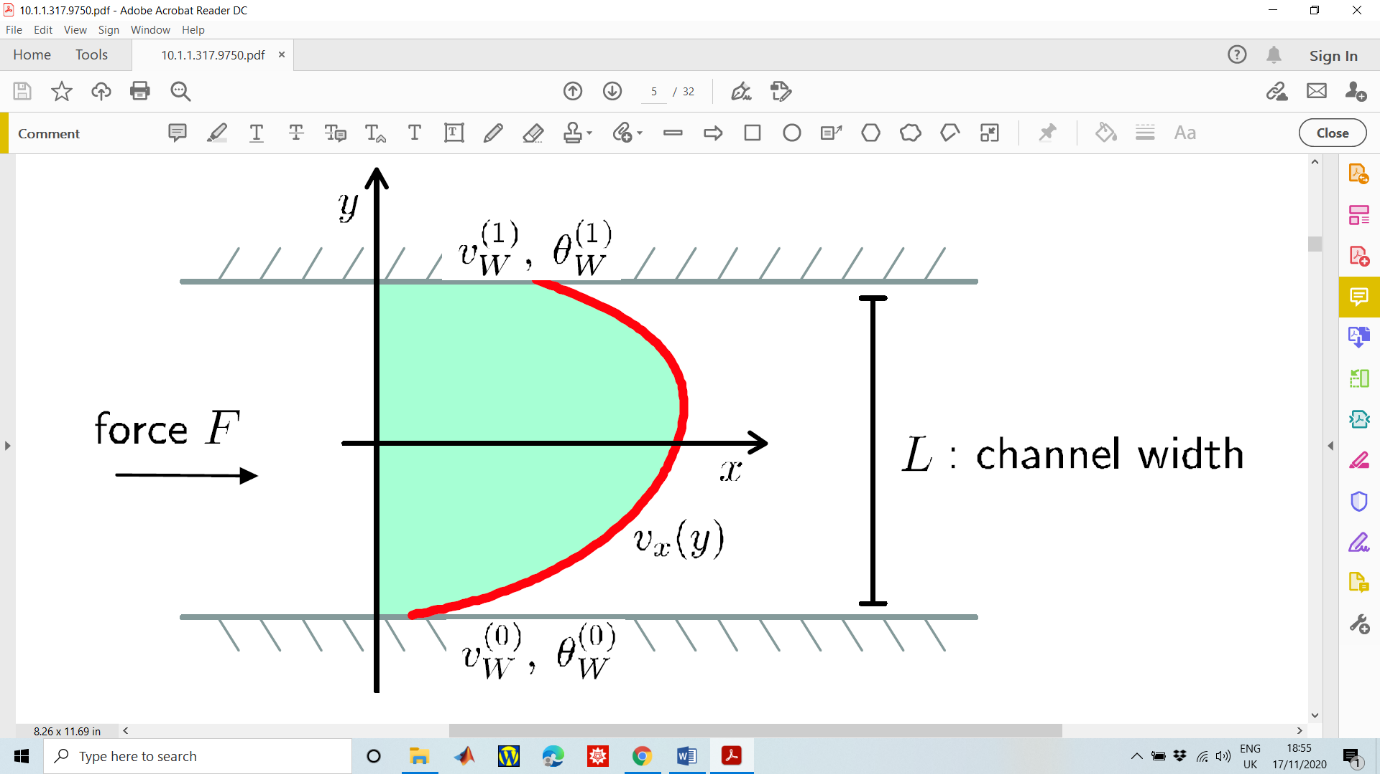
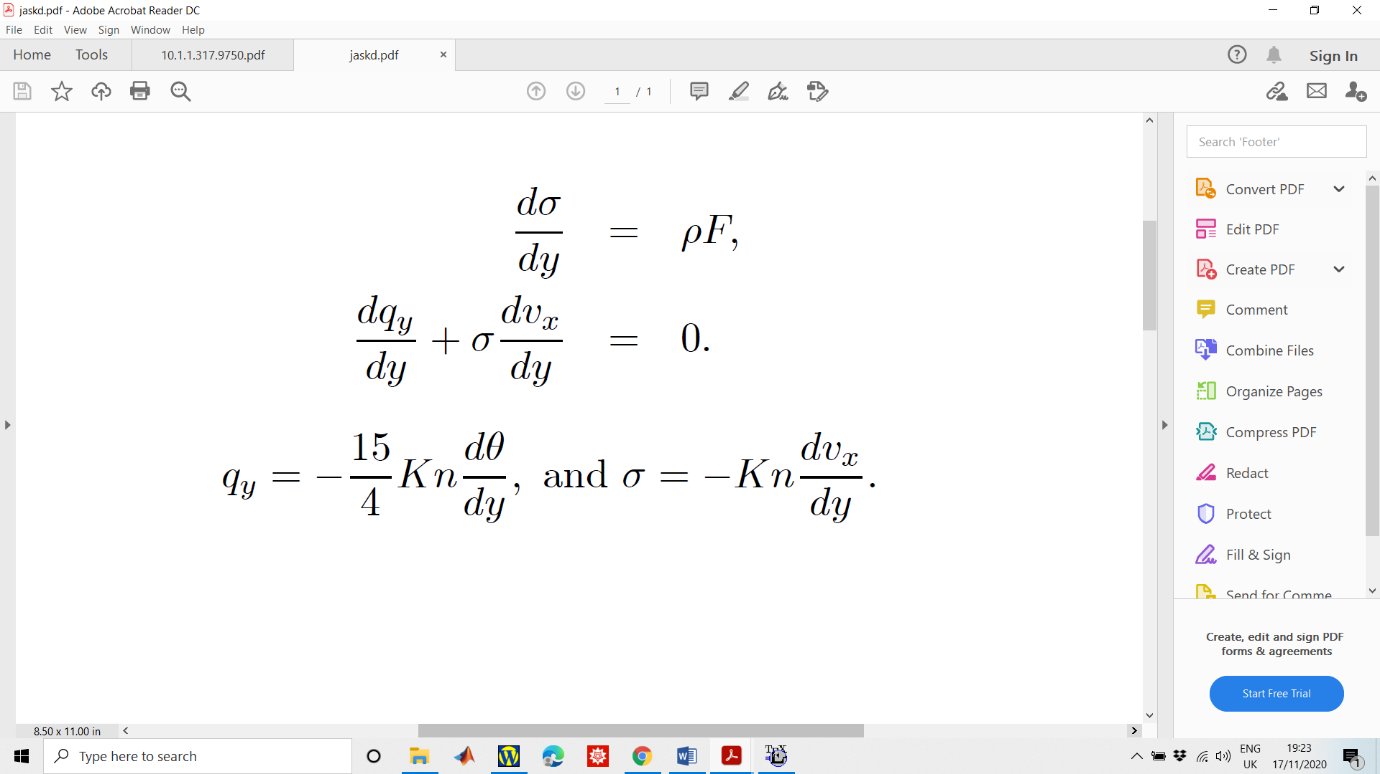
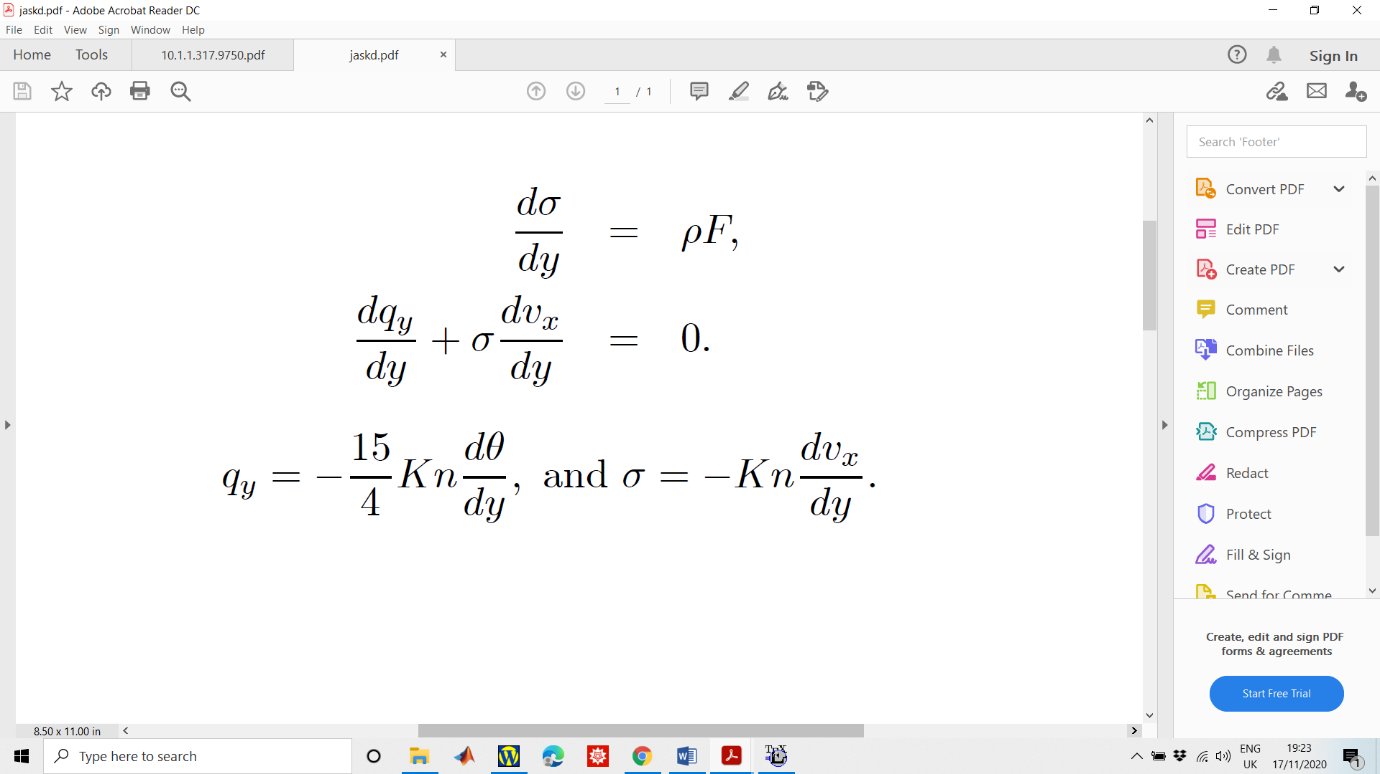


Fig. 1: General shear flow setting. The gas flows between infinite plates with velocities and temperature. The force is given by gravity or a pressure gradient.

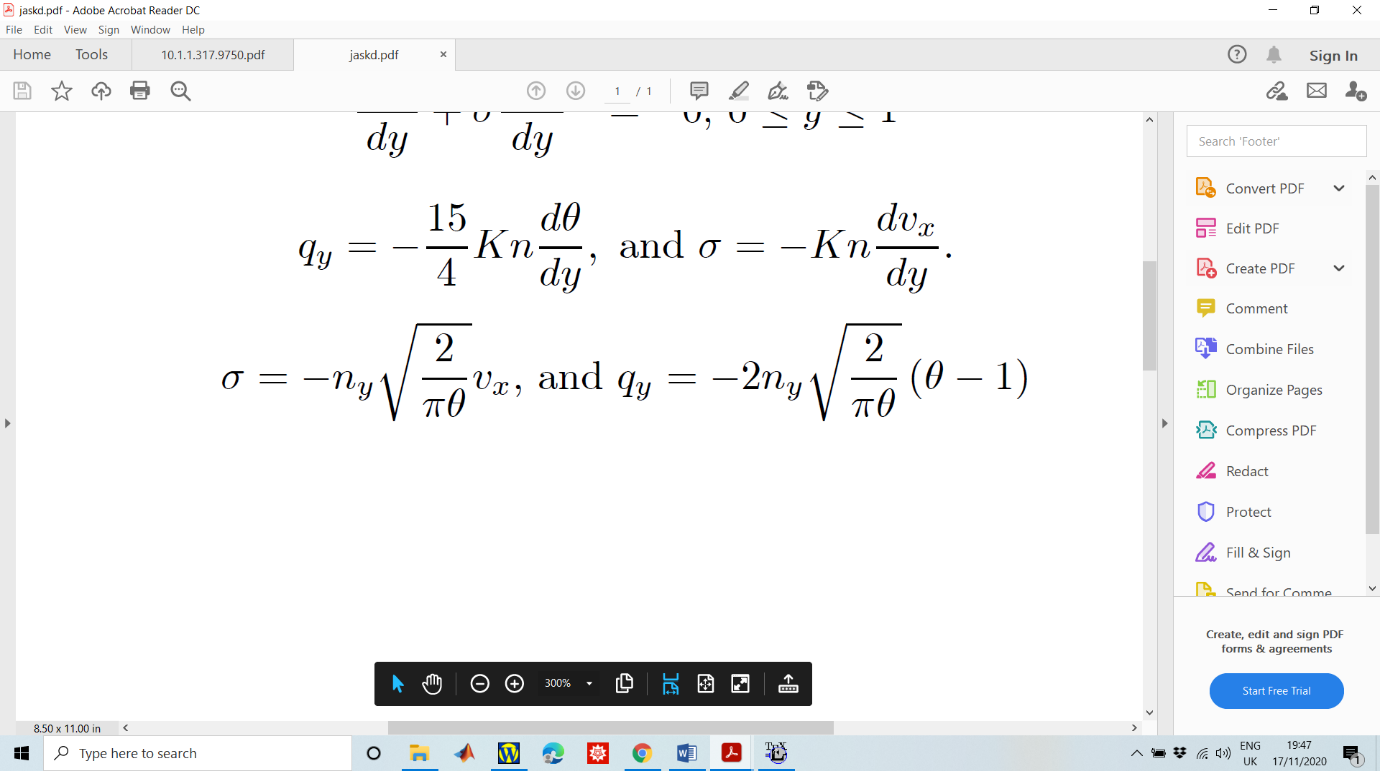
The differential equations, describing this process are given by the conservation laws:

Here*,*  is the velocity of the gas in x-direction, is gas density, which is given by the ideal gas law, where is the dimensionless pressure (a constant) in the gas across the channel and is the dimensional temperature of the gas.

The heat flux in direction and the shear stress are given by the Fourier’s law and the Navier-Stokes relations, respectively as



Here, is defined as the Knudsen number, a parameter which dictates the degree of rarefaction in the gas.

This system of four ODEs needs to be solved for four unknowns (). The required boundary conditions for such systems are given by the velocity-slip and temperature-jump boundary conditions, as

Where following the setting of Fig.1 these boundary conditions have to hold on both sides of the channel with for lower () and upper wall (), respectively.

Our task is to use the **mid-point finite difference method** in order to solve the above system of BVPs with, and with Knudsen number along with discretized points.

**Part 1**

(a) Plot velocity vs for in the same plot.

(b) Plot velocity vs for in the same plot.

(c) Plot velocity vs for in the same plot.

(d) Plot velocity vs for in the same plot.

**A picture containing map, smoke, flying, group

Description automatically generatedINSERT FIGURE A HERE**

**A close up of a map

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**Chart

Description automatically generatedINSERT FIGURE C HERE**

**Chart

Description automatically generatedINSERT FIGURE D HERE**

**Part 2**

Perform an empirical error of convergence (EOC) analysis of the numerical method in velocity, with

**INSERT FIGURE EOC HERE**

**Chart, line chart

Description automatically generated**

**A picture containing chart

Description automatically generatedKn = 0.068, Slope of best fit line = 2.0304**

**A picture containing graphical user interface

Description automatically generatedKn = 0.5, Slope of best fit line = 2.0292**

How the EOC is affected by the Knudsen number?

**Ans:** Larger Knudsen number (i.e. ) shows smaller error in . This might be because and as increases, tends to 0 i.e. tends towards being a constant. This maybe the reason why error decreases. For both and , the method is 2nd order as we can see from the slope of the best fit line which is in case of and in case of . This can be seen from **Figure A** in part **A.** There is no change in order of the method as changes.